A Real Coded Vector Population-Based Incremental Learning for Multi-objective Optimizations of Electromagnetic Devices

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Nowadays, the evolutionary algorithm (EA) has become the standard and paradigm for solving multi-objective design problems. The shortcomings when implementing the genetic operations in EA is however limiting the popularity of EA in engineering applications. There are also only lukewarm efforts devoted to address the inadequacy of EAs in abstracting the characteristic features of the landscape of an objective function. Consequently, increasing attentions have now turned to EAs based on Probabilistic Models (EAPM) in computational intelligence studies. In this regard, a real coded scalar population-based incremental learning algorithm, an EAPM, is extended for multi-objective optimizations of electromagnetic devices. Major improvements include the design of a generating mechanism for new intermediate solutions, the selection of *elite* solutions to update the probability matrix, and matrix updating formulations. Numerical results on a high frequency inverse problem are reported to showcase the merits of the proposed algorithm.

***Index Terms*— Design optimization, inverse problem, multi-objective, population-based incremental learning.**

I. A Real Coded Vector PBIL

In design optimization of an electromagnetic device, it is often required to satisfy several seemingly conflicting criteria (objectives). These type of problems are generally called multi-objective or vector optimization problems, and their final solutions are sets of compromises of different objectives commonly referred as Pareto optimal solutions. An acceptable multi-objective optimizer should have the ability to find as many Pareto optimal solutions as possible, and these solutions should also be as uniformly distributed as possible. To achieve these two ultimate goals, a huge amount of efforts have been devoted to the advancement of evolutionary algorithms (EA), because of their suitability in finding multiple solutions in a single run [1]-[3]. Nowadays, EAs have become the paradigms for solving multi-objective design problems.

The components of EAs, such as population size, variation operators, parent selection, reproduction and inheritance, survival competition methods, among others, must however be designed properly before one can secure a good balance between exploration and exploitation searches in solving multi-objective problems [4]. To circumvent these issues, increasing attentions are now being turned to EAs based on Probabilistic Models (EAPM) in computational intelligence studies [5].

The Population Based Incremental Learning (PBIL) algorithm, developed by combining genetic algorithm (GA) and competitive learning strategy [6], is one of the earliest EAPMs. It excludes the use of traditional crossover and mutation operators of other EAs. Furthermore, a salient characteristic of PBIL algorithm is that it can explicitly extract the global statistical information from its previous searches to build a probability distribution model of promising solutions to bias the subsequent searches towards the finding of improved solutions.

To extract the statistical information and the behavior of the objective functions from its previous searches, a PBIL uses a real valued probability vector to learn from the *elite* solutions being searched so far. In essence, the PBIL was originally implemented as a binary coded metaheuristic [6], and extended to real coded variants using either an interval approach [7] or a histogram [8]. However, the boundary movement for the former or the generation of feasible solutions for the latter is, to some extent, deterministic, which deviates from the stochastic nature of the original algorithm. In this regard, a novel real coded scalar PBIL is proposed in [9]. However, lukewarm efforts have been devoted to PBILs, especially the real coded vector PBILs, in the study of design optimizations in electromagnetics. In this paper, the real coded scalar PBIL is improved and extended to multi-objective design optimization applications in an attempt to promote interests among fellow researchers.

As an illustration, one considers a minimizing problem. The counterpart of the real valued probability vector in binary PBILs for the proposed real coded one is a probability matrix P. To define the probability matrix P, the decision variable in each dimension is equally divided into M sub-intervals. For example, the j^{th} interval of the i^{th} variable x_i is defined as $u_{ij}=[a_i+(j-1)\Delta, a_i+j\Delta](\Delta=(b_i-a_i)/M)$. The value, p_{ij}, of the probability matrix P presents the probability of x_i in the interval u_{ij}. Initially, all the values of the probability matrix are set to $1/M$ to ensure that the sampling from this probability matrix will produce a uniform distribution of the initial population in the feasible parameter space. In the searching process, the probability matrix P is used to generate the whole population in such a way that the probability of x_i within the interval u_{ij} is proportional to p_{ij}. To generate intermediate solutions which concentrate on the center of a sub-interval, the normal distribution function, instead of the existing uniform distribution, is used; and a typical implementation of such a generating mechanism is explained in Fig.1. In Fig. 1, $\boldsymbol{x}$ is the vector of the decision variables; a_i and b_i are, respectively, the lower and upper bounds of the i^{th} variable x_i; $N(u_{ik},\sigma_{ik}^2)$ is the normal distribution function with its mean μ_{ik} and standard deviation σ_{ik}.

To abstract the stochastic information about the objective functions in order to guide the search toward promising regions, the probability matrix P is updated using the so far searched best solution, $\boldsymbol{x}^{best}$, of the population in a scalar PBIL. However, the best solutions of a multi-objective design problem are not unique if the objective functions are equally treated. This will give a rise to a dilemma in the selection of the best solution so far searched in a vector PBIL. In order to bring a good balance between exploitation and exploration searches, every individual memories its own 'Pareto optimal' solutions, the so far searched best solution, of the individual in issue, and selects randomly one from them when updating its probability distribution. Moreover, the probability matrix is updated at the end of every population by using

$$\begin{aligned} LR_{ij} &= LR_0 \exp(-(j-r)^2)/(b_i - a_i) \\ p_{ij}(t+1) &= LR_{ij} + (1 - LR_i)\, p_{ij}(t) \\ p_{ij}(t+1) &= p_{ij}(t+1) / (\sum_{j=1}^{M} p_{ij}(t+1)) \end{aligned} \tag{1}$$

where, LR_0 is the initial (positive) learning rate, r is the interval index which contains x_i^{best}.

Procedures to generate a population using
a probability matrix P
For m=1 to N
  For i=1 to n
    r=rand(0,1)
    For k=1 to M
      If $\sum_{l=0}^{k-1} p_{il} \le r \le \sum_{l=0}^{k} p_{il}$ (p_{i0}=0) then
        $x_i = N(u_{ik}, \sigma_{ik}^2)$
      Endif
    End
  End
  $\boldsymbol{x}_m=(x_1, x_2 \ldots, x_n)$
End

Fig. 1 Generating a population using the probability matrix P

Other approaches designed for extending scalar PBIL to vector one as well as the implemental details will be given in the full paper because of space limitations in this abstract.

II. Numerical Examples and Conclusions

To validate the proposed vector PBIL algorithm, it is numerically experimented on several design optimizations in both high and low frequency regimes. Due to space limitations, only numerical results on an antenna array design are reported.

In this case study, the desired field pattern of a shaped beam with a cosecant variation is reconstructed using a non-uniformly spaced antenna array. The desired pattern is defined as [10]: the field will vary following a cosecant function in the interval $\cos\theta \in [0.1, 0.5]$ having a Maximum SideLobe Level (MSLL) smaller than -22 dB in the residual intervals.

To produce a radiation pattern which is as close as possible to this desired one, and which simultaneously has a minimum possible SideLobe Level as small as possible, a two objective optimization problem is formulated [10] and resolved.

To quantitatively evaluate the performances of a vector optimal algorithm in the following discussions, the *convergence* measure γ is used as the metric to measure the convergence of the obtained solution set to a known set of Pareto solutions, while the displacement metric is employed to gauge the uniformity or diversity performance of the attained solutions over the non-dominated front [11]. If the set which consists of uniformly spaced true Pareto solutions obtained using an exhaustive approach in this case study is P^*; and the set of the final solution of the vector optimizer is Q, then the predefined two metrics are given as

$$\gamma = \sum_{i=1}^{|Q|} \Big(\min_{j\in[1,|P^*|]} (d(i,j)) \Big/ |Q| \tag{2}$$

$$displacement = \sum_{i=1}^{|P^*|} \Big(\min_{j\in[1,|Q|]} (d(i,j)) \Big/ |P^*| \tag{3}$$

where $d(\cdot,\cdot)$ is the Euclidean distance of the two set solutions.

In this case study, a 19-element non-uniform antenna array is optimized. For performance comparisons, this case study is solved by using the proposed algorithm and the improved vector tabu search method [10]. Moreover, to evaluate the average performance of an algorithm, each algorithm is run randomly and independently 15 times. The average performances of the 15 runs for the two algorithms are summarized as: (1) The average numbers of iterations for the proposed and the vector tabu search algorithms are, respectively, 28996 and 29465; (2) The parameters γ and *displacement* of the final solutions for the proposed algorithm are, respectively, 0.000018 and 0.00026; while the metrics γ and *displacement* of the final solutions for the improved tabu search method are, respectively, 0.000018 and 0.00029.

In short, the proposed vector PBIL algorithm outperforms the well developed vector tabu search method in terms of both solution speed and stochastic performances.

References

[1] Jan K. Sykulski, "State of the art and new challenges in design optimisation of electromagnetic devices," *Lecture of ICEF2016*, September 18-20, 2016.

[2] Ray S., Lowther D.A.,"Multi-objective optimization applied to the matching of a specified torque-speed curve for an internal permanent magnet motor," *IEEE Trans. Magn.*, vol.45, pp. 1518-1521, Mar. 2009.

[3] P. Di Barba, F. Dughiero, M. E. Mognaschi, A. Savini, S. Wiak," Biogeography-inspired multiobjective optimization and MEMS design," *IEEE Trans. Magn.*, vol.52, article ID: 7201504, 2016.

[4] Han, K. H., and Kim, J. H., "Quantum-inspired evolutionary algorithm for a class of combinatorial optimization," *IEEE Trans. Evol. Comput.*, vol. 6, No. 6, Dec. 2002, pp. 580-593.

[5] Lozano, J. A., Zhang, Q., and Larranaga, P., "Guest editorial: special issue on evolutionary algorithm based on probabilistic models," *IEEE Trans. Evol. Comput.*, vol. 13, No. 6, Dec. 2009, pp. 1197-1198.

[6] S. Baluja and R. Caruana, "Removing the genetics from the standard genetic algorithm," *Proc. of ICML95*, 1995, pp. 38-46.

[7] Bo Yuan, Gallagher, M., "Playing in continuous spaces: some analysis and extension of population-based incremental learning," *The 2003 Congress on Evolutionary Computation (CEC '03)*, 2003, vol. 1, pp. 443-450.

[8] Sujin Bureerat, "Improved population-based incremental learning in continuous spaces," *Advances in Intelligent and Soft Computing*, vol. 96, 2011, pp 77-86.

[9] Siu Lau Ho, Linhang Zhu, Shiyou Yang, and Jin Huang," A real coded population-based incremental learning for inverse problems in continuous space," *IEEE Trans. Magn.*, vol.52, article ID: 7000904, 2016.

[10] Jiaqiang Yang, Shiyou Yang, and Peihong Ni, " A vector tabu search algorithm with enhanced searching ability for Pareto solutions and its application to multiobjective optimizations," *IEEE Trans. Magn.*, vol.52, article ID: 7001204, 2016.

[11] S. Bandyopadhyay, S. Saha, U. Maulik, and K. Deb, "A simulated annealing-based multiobjective optimization algorithm: AMOSA," *IEEE Trans. Evol. Comput.*, vol. 12, pp. 269-283, 2008.